Migration of vesicles and flexible fibers in Poiseuille flow

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Soft Matter 2016
The system

- unbounded Poiseuille flow

\[ \mathbf{v}_\infty = \left( \alpha y^2 - v_m \right) \mathbf{e}_x \]

- vesicle or flexible fiber, initially aligned with the flow
Question

- Are there any universal features of the dynamics of vesicles and flexible fibers?
Vesicle

- Membrane, fluid inside, $\lambda = \eta_{\text{in}}/\eta$
- Constant volume $V$
  (radius $R_0 = \left(\frac{3V}{4\pi}\right)^{1/3}$ of the sphere with the same volume)
- Constant membrane surface area $S$ (reduced volume $\nu = 6\sqrt{\pi}S^{-3/2}V$)
- Bending rigidity modulus $\kappa$ and capillary number, $C_a = \alpha\eta R_0^4/\kappa$
Flexible fiber

- A chain of close solid beads
- Elastic and bending forces between the consecutive beads
- At the equilibrium the fiber is straight
- Very large value of the spring constant
- Different values of the bending stiffness $A = Y/(625\pi \alpha \eta d)$
Methods of solving the Stokes equations

- Stokes equations
- Green tensors
- Boundary integral equations
- Boundary element method for vesicles
- Multipole expansion corrected for lubrication for fibers
Cross-stream migration and accumulation

0 - central plane of the unbounded Poiseuille flow
Matching geometry of vesicles and fibers

Equilibrium shapes

Vesicle: $\lambda = 12$, $C_a = 0.08$, $\nu = 0.6$. 
Lateral migration and accumulation

Distance from the central plane versus time

Vesicles with $C_a=0.08$

Fibers with $A=0.024$
How the accumulation position depends on $A$ and $C_a$?
Scaling: local shear rate approximation

\[ \frac{y - y_0}{y_0} \ll 1, \]

we approximate the Poiseuille flow by the local shear,

\[ v_\infty(y) \approx 2\alpha y_0(y - y_0) + (\alpha y_0^2 - v_m). \]

\[ \alpha \rightarrow \alpha/\beta \quad \Rightarrow \quad 1/C_a \rightarrow \beta/C_a \quad \text{and} \quad A \rightarrow \beta A. \]

The same essential dynamics is obtained for the same local shear rate, what corresponds to the rescaled center position \( y_0 \),

\[ 2\alpha y_0 \rightarrow 2\alpha y_0 \quad \Rightarrow \quad y_0 \rightarrow \beta y_0. \]

Conclusion:

Far from the central plane of the flow,

\[ y_c \sim 1/C_a, \quad y_c \sim A. \]
Numerical results

\[ y_c/R_0 \]

vs.

\[ C_a^{-1} \]

and

\[ y_c/R_0 \]

vs.

\[ A \]
The matching

\[ \frac{1}{C_a} \simeq 520A. \]
The relevant parameter

The capillary number related to the local shear rate,

\[ C_{a,s} = 2C_y y_0 / R_0. \]
Tumbling frequency

![Graph showing tumbling frequency vs. concentration](image)

Vesicles

![Graph showing tumbling frequency vs. concentration](image)

Fibers
Migration velocity

\[ \frac{V_{\text{mig}}}{(\alpha R_0^2)} \]

- **Vesicles**
- **Fibers**
Snapshots from the evolution of shapes ($C_a = 0.15$)

$y_0/R_0 = 9.3$
$C_{a,s} = 2.79$

$y_0/R_0 = 4.0$
$C_{a,s} = 1.20$
Snapshots from the evolution of shapes ($C_a = 0.15$)

$\frac{y_0}{R_0} = 9.3$

$C_a,s = 2.79$

$\frac{y_0}{R_0} = 4.0$

$C_a,s = 1.20$

Snapshots from the evolution of shapes ($C_a = 0.01$)

$\frac{y_0}{R_0} = 139.5$

$C_a,s = 2.79$

$\frac{y_0}{R_0} = 60$

$C_a,s = 1.20$
C-shapes are due to the flow curvature

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$y_0/R_0$: 9.3 4.0 139.5 60
$C_{a,s}$: 2.79 1.20 2.79 1.20
Shapes at the flipping instant

**xy-plane**

- $C_{a,s} = 0.8$
- $C_{a,s} = 1.0$
- $C_{a,s} = 1.2$
- $C_{a,s} = 1.4$
- $C_{a,s} = 1.6$
- $C_{a,s} = 1.8$
- $C_{a,s} = 2.0$
- $C_{a,s} = 2.4$

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- Scaling laws: the local shear rate times the bending stiffness is the essential parameter
- C-shapes close to the central plane
- Accumulation mechanism: deformation of shapes (still to be understood)
Are there differences in shapes between inward and outward migration?

\[ C_a \approx 0.15. \text{ Consecutive frames are separated by } \delta t = 0.02\tau \]