NON-NEWTONIAN TRANSPORT PROPERTIES OF GRANULAR SUSPENSIONS UNDER SIMPLE SHEAR FLOW

SPhinX
STATISTICAL PHYSICS in EXTREMADURA

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OUTLINE

- BOLTZMANN KINETIC EQUATION FOR GRANULAR SUSPENSIONS

- SIMPLE (UNIFORM) SHEAR FLOW (**USF**)

- RHEOLOGICAL PROPERTIES: GRAD’S MOMENT METHOD AND BGK-LIKE KINETIC MODEL

- COMPARISON WITH MONTE CARLO SIMULATIONS

BOLTZMANN KINETIC EQUATION

• Rapid granular flows ⟶ Fluid of hard spheres with *inelastic* collisions

• Simplest model: *smooth* hard spheres. Inelasticity characterized by a constant coefficient of normal restitution

$$0 \cdot \ \alpha \cdot \ 1$$

• Kinetic level description: one-particle velocity distribution function

$$f(\mathbf{r}, \mathbf{v}, t)$$

Provides all the relevant information on the state of the system

• At low density: *inelastic* Boltzmann equation

In most of the applications the effect of interstitial gas on grains is neglected. Here, we consider a gas-solid interaction force modeled via a generalized Langevin term:

$$m\frac{d\mathbf{v}}{dt}=-\beta(\mathbf{U}-\mathbf{U}_g)-\gamma\mathbf{V}+\mathbf{F}_{\text{st}}$$

$$\mathbf{V}=\mathbf{v}-\mathbf{U}$$

Boltzmann kinetic equation for granular suspensions

$$\partial_t f+\mathbf{v}\cdot\nabla f-\frac{\beta}{m}\Delta\mathbf{U}\cdot\frac{\partial f}{\partial\mathbf{V}}-\frac{\gamma}{m}\frac{\partial}{\partial\mathbf{V}}\cdot\mathbf{V}f-\frac{1}{2}\xi\frac{\partial^2 f}{\partial V^2}=J[f,f]$$

$$\Delta U=\mathbf{U}-\mathbf{U}_g$$

(Garzó, Tenetti, Subramaniam, Hrenya, JFM **712**, 129 (2012))

$(\beta, \gamma, \xi) \longrightarrow$ Parameters of the model

In particular, $\xi \propto |\Delta \mathbf{U}|^2$

Dilute suspensions and at low Reynolds numbers

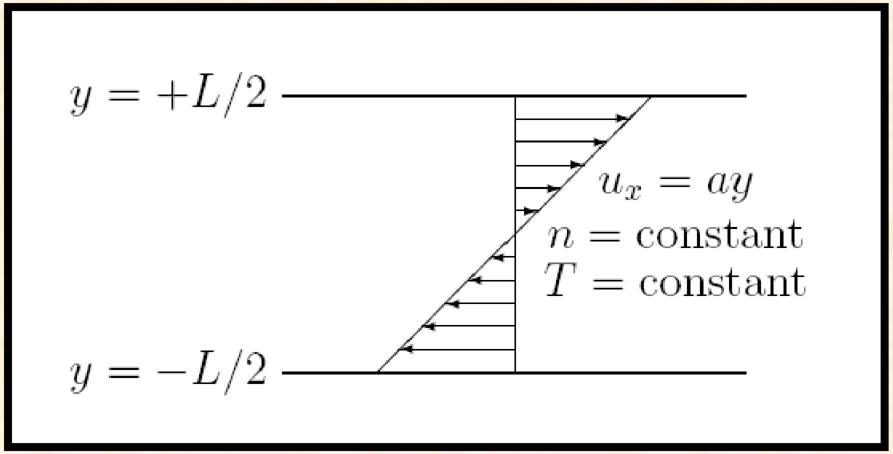
$$\gamma = 3\pi\mu_g \sigma R(\phi)$$

$$R(\phi) = 1 + 3\sqrt{\frac{\phi}{2}}$$

UNIFORM SHEAR FLOW (USF)

- Due to the kinetic energy dissipation in collisions, energy must be externally injected to achieve stationary conditions.

- Mechanism of energy input: Simple shear flow



$$\mathbf{U} = \mathbf{U}_g$$

•USF becomes *spatially homogeneous* in the local Lagrangian frame moving with the flow velocity: $f(\mathbf{r}, \mathbf{v}, t) \to f(\mathbf{V}, t)$

In the ***steady*** state

$$-aV_y\frac{\partial f}{\partial V_x}-\frac{\gamma}{m}\frac{\partial}{\partial \mathbf{V}}\cdot\mathbf{V}f=J[f,f]$$

Equation for the granular temperature

$$-\frac{2}{dn}P_{xy}a=\frac{2T}{m}\gamma+\zeta T$$

$$\zeta\propto 1-\alpha^2$$

Viscous heating term is exactly compensated by the cooling terms (collisional dissipation plus viscous friction term). Intrinsic connection between the velocity gradient (nonequilibrium parameter), the coefficient of restitution and the friction coefficient

Pressure tensor $$P_{ij} = \int d\mathbf{V}\, mV_iV_j\, f(\mathbf{V})$$

$$a^*(\alpha, \gamma^*) = \frac{a}{\nu(T)}, \quad \nu(T) \propto \sqrt{T}$$

$$\gamma^* = \frac{\gamma}{m\nu}$$

Collision frequency for hard spheres

Relevant dimensionless number in suspensions is the Stokes number St:

$$\mathrm{St} = \frac{a^*}{\gamma^*/R(\phi)}$$

- Rheological properties: Pressure tensor elements

$$a_{ik}P_{kj} + a_{jk}P_{ki} + \frac{2\gamma}{m}P_{ij} = \int d\mathbf{V}\; mV_iV_jJ[f,f] \equiv \Lambda_{ij} \qquad a_{ij} = a\delta_{ix}\delta_{jy}$$

Closed problem once the Boltzmann collisional moment Λ_{ij} is known. This requires the explicit knowledge of the distribution function. Formidable task!!

We estimate this collisional moment by using two different (but complementary) theoretical approaches.

1. Grad's moment method

(H. Grad, Commun. Pure Appl. Math. **2**, 331 (1949))

$$f(\mathbf{V}) \to f_{\mathrm{M}}(\mathbf{V}) \left(1 + \frac{m}{2nT^2} V_i V_j \Pi_{ij}\right)$$

$$\Pi_{ij} = P_{ij} - p\delta_{ij}, \quad p = nT$$

For spheres (d=3)

$$\Lambda_{ij} = -p\nu(1+\alpha) \left[\frac{5}{12}(1-\alpha)\delta_{ij} + \frac{3-\alpha}{4}\left(\Pi_{ij}^* + \frac{1}{14}\Pi_{ik}^*\Pi_{kj}^*\right) - \frac{5+3\alpha}{672}\Pi_{k\ell}^*\Pi_{k\ell}^*\delta_{ij}\right]$$

$$\Pi_{ij}^* = \Pi_{ij}/p$$

In contrast to previous attempts (Tsao&Koch, JFM 1995; Sangani *et al.*, JFM 1996), we retain nonlinear terms in the pressure tensor in the evaluation of Λ_{ij}

This refinement allows us to obtain nonzero normal stress differences which are observed in computer simulations.

2. BGK-type kinetic model of the Boltzmann equation

(Brey, Dufty, Santos, JSP **97**, 281 (1999))

$$J[f,f] \to -\chi(\alpha)\nu\left(f - f_{\mathrm{M}}\right) + \frac{\zeta_0}{2}\frac{\partial}{\partial \mathbf{V}}\cdot \mathbf{V} f$$

free parameter of the model

An exact solution is obtained. In particular, if

$$\chi = \frac{1+\alpha}{2}\left[1 - \frac{d-1}{2d}(1-\alpha)\right]$$

the BGK results for the pressure tensor agree with those derived from linear Grad's solution.

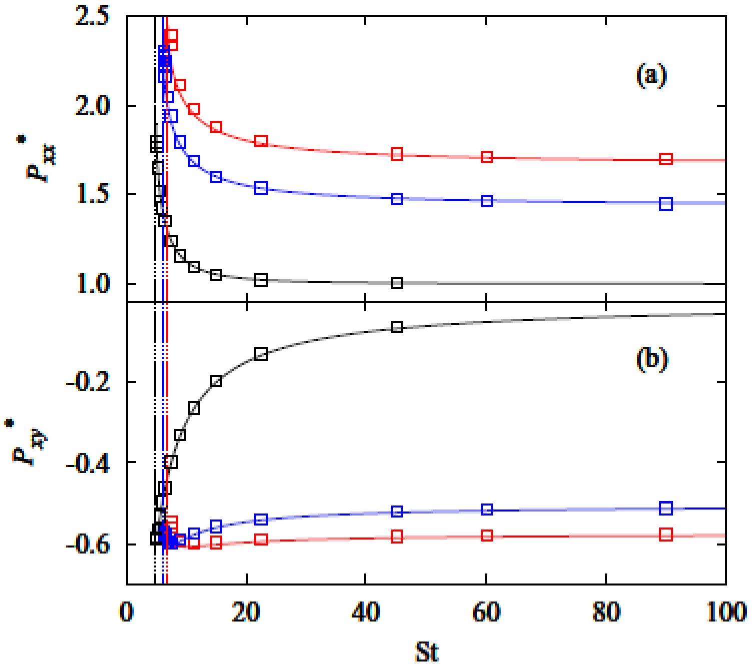


FIG. 3. (Color online) Dependence of the (reduced) elements of the pressure tensor P_{xx}^* (a) and P_{xy}^* (b) on the Stokes number St for several values of the coefficient of restitution α: $\alpha = 1$ (black), $\alpha = 0.7$ (blue), and $\alpha = 0.5$ (red). The solid lines are the theoretical results obtained from nonlinear Grad's solution while the symbols refer to the results obtained from Monte Carlo simulations. We have marked as vertical dotted lines the minimum allowed value for the Stokes number St.

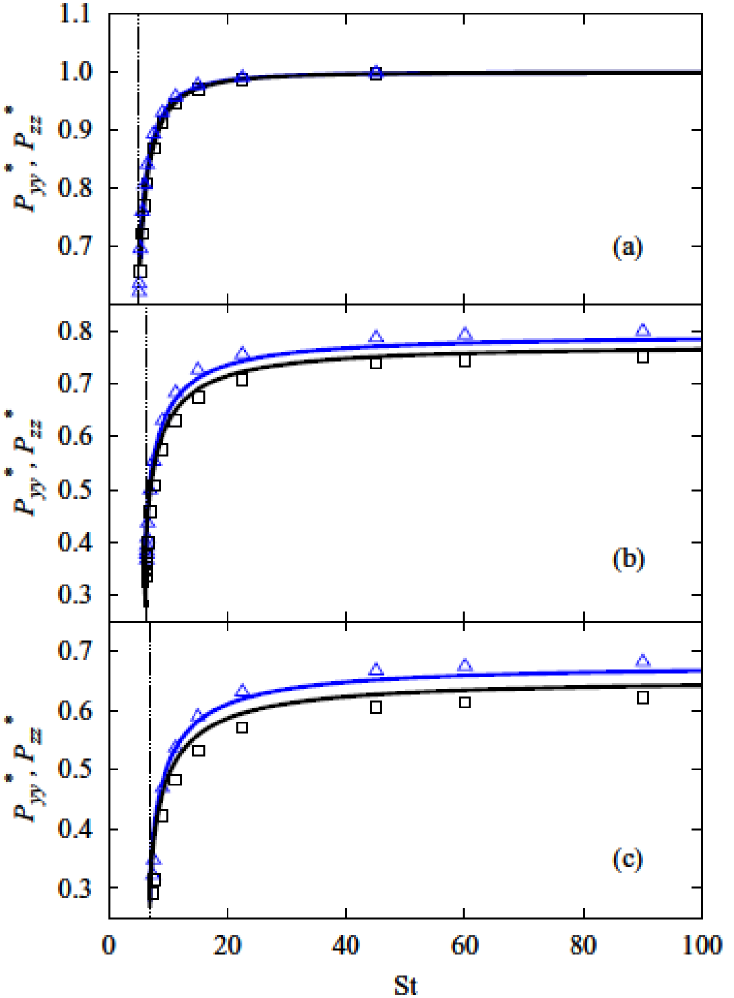


FIG. 4. (Color online) Dependence of the (reduced) diagonal elements of the pressure tensor P_{yy}^* (black lines and squares) and P_{zz}^* (blue lines and triangles) on the Stokes number St for several values of the coefficient of restitution α: $\alpha = 1$ (a), $\alpha = 0.7$ (b), and $\alpha = 0.5$ (c). The solid lines are the theoretical results obtained from nonlinear Grad's solution while the symbols refer to the results obtained from Monte Carlo simulations. As in Fig. 3, we have marked as vertical dotted lines the minimum allowed value of the Stokes number St for each value of α.

(Chamorro, Vega Reyes, Garzó, PRE **92**, 052205(2015))

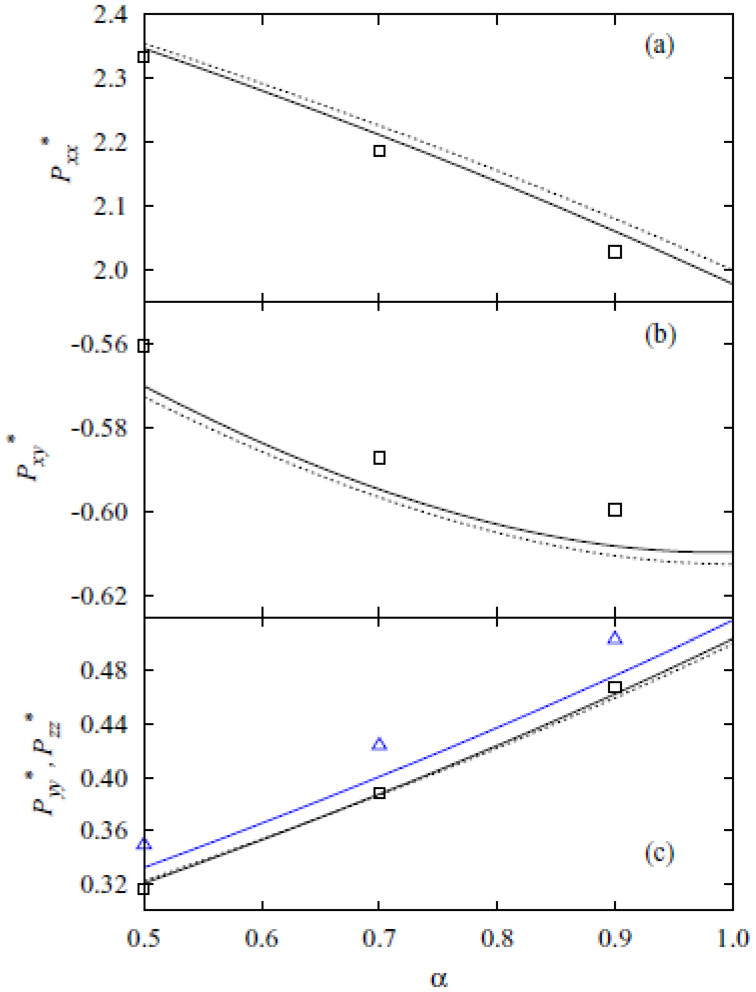


FIG. 5. (Color online) Plot of the (reduced) nonzero elements of the pressure tensor P_{xx}^* (a), P_{xy}^* (b), P_{yy}^* and P_{zz}^* (c) as functions of the coefficient of restitution α for $\gamma^* = 0.5$. The solid and dotted lines correspond to the results obtained from nonlinear and linear Grad's solution, respectively. Symbols refer to Monte Carlo simulations. In (c), the blue solid line and triangles are for the element P_{zz}^* while the black solid line and squares are for the element P_{yy}^*. Note that linear Grad's solution (dotted line) yields $P_{yy}^* = P_{zz}^*$.

BGK results

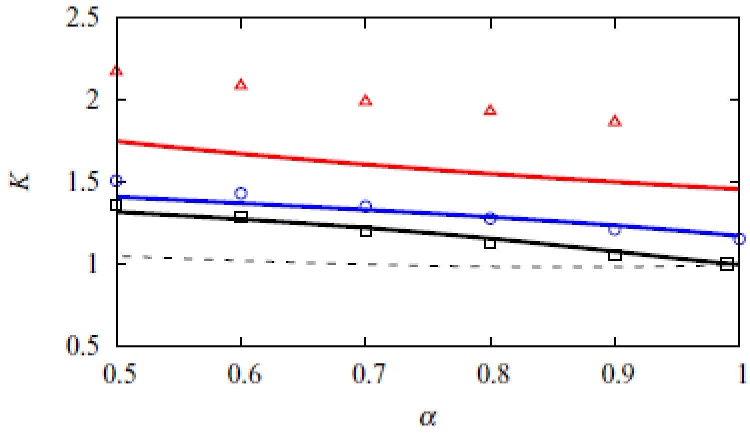


FIG. 6. (Color online) Plot of the kurtosis $K \equiv \langle V^4 \rangle / \langle V^4 \rangle_0$ vs the coefficient of restitution α for three different values of the (reduced) friction coefficient γ^*: $\gamma^* = 0$ (black line and squares), $\gamma^* = 0.1$ (blue line and circles), and $\gamma^* = 0.5$ (red line and triangles). The solid lines correspond to the results obtained from the BGK-type model while symbols refer to DSMC results. The dashed line is the result obtained in Ref. [35] for the homogeneous cooling state.

Steady scaled temperature

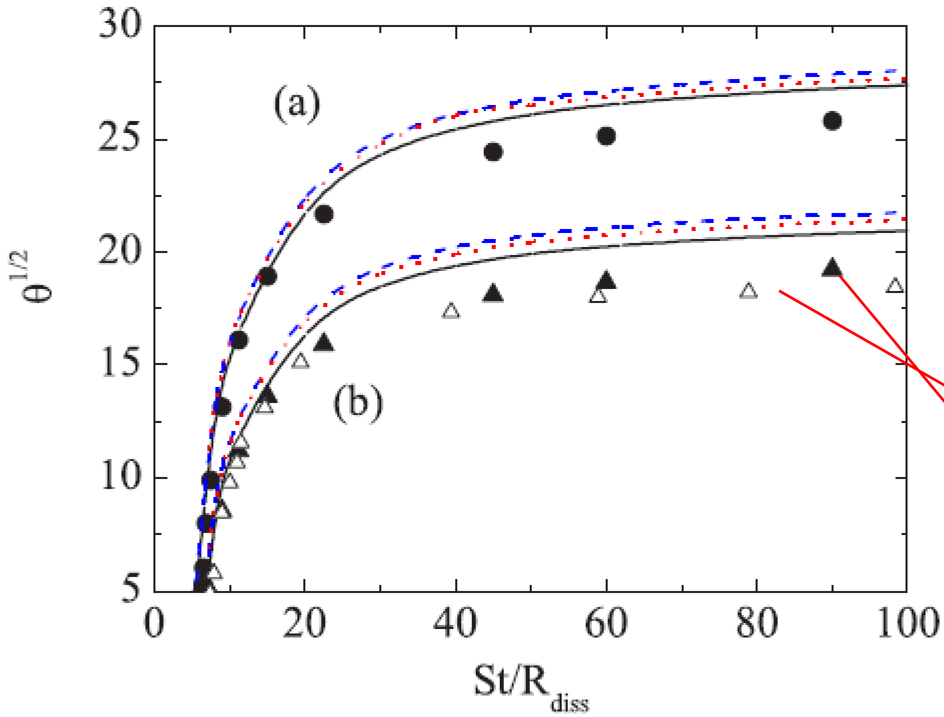


FIG. 9. (Color online) Plot of the square root of the steady granular temperature $\theta^{1/2}$ as a function of $\mathrm{St}/R_{\mathrm{diss}}$ in the case of hard spheres ($d=3$) for $\phi=0.01$. Two different values of the coefficient of restitution have been considered: $\alpha=0.7$ (a) and $\alpha=0.5$ (b). The solid line is the Grad solution (including nonlinear contributions) to the Boltzmann equation, the dashed (blue) line corresponds to the BGK results (which coincide with those obtained from the linear Grad solution), and the dotted (red) line refers to the results obtained by Sangani *et al.* [32] from the Enskog equation by applying (linear) Grad's method. The black circles and triangles are the simulation results obtained here by means of the DSMC method for $\alpha=0.7$ and $\alpha=0.5$, respectively, while the empty triangles are the results obtained in Ref. [32].

$$\theta = \frac{4T}{m\sigma^2 a^2}$$

Sangani *et al.*, JFM 1996
(dense fluid)

DSMC, dilute gas

BGK-velocity distribution function

$$f(\mathbf{V}) = n\left(\frac{m}{2T}\right)^{d/2}\varphi(\mathbf{c}), \quad \mathbf{c} \equiv (m/2T)^{1/2}\mathbf{V}.$$

Scaled distribution (spheres):

$$\varphi(\mathbf{c}) = \pi^{-d/2}\int_0^\infty \mathrm{d}t\, e^{-(1-d\widetilde{\epsilon})t}\, \exp\left[-e^{2\widetilde{\epsilon}t}\left(\mathbf{c}+t\widetilde{\mathbf{a}}\cdot\mathbf{c}\right)^2\right]$$

$$\widetilde{\epsilon} \equiv \frac{\gamma^* + \zeta_0^*/2}{\chi}, \quad \widetilde{a}_{ij} \equiv a_{ij}^*/\chi.$$

Marginal distributions:

$$\varphi_x^{(+)}(c_x) = \int_0^\infty dc_y \int_{-\infty}^\infty dc_z \; \varphi(\mathbf{c})$$

$$\varphi_y^{(+)}(c_y) = \int_0^\infty dc_x \int_{-\infty}^\infty dc_z \; \varphi(\mathbf{c})$$

Maximum-entropy formalism (Jenkins&Richman, JFM 1988)

$$f(\mathbf{V}) = n\pi^{-3/2} \det(\mathsf{Q})^{1/2} \exp\left(-\mathbf{V}\cdot\mathsf{Q}\cdot\mathbf{V}\right), \quad \mathsf{Q} = \frac{1}{2}mn\mathsf{P}^{-1}$$

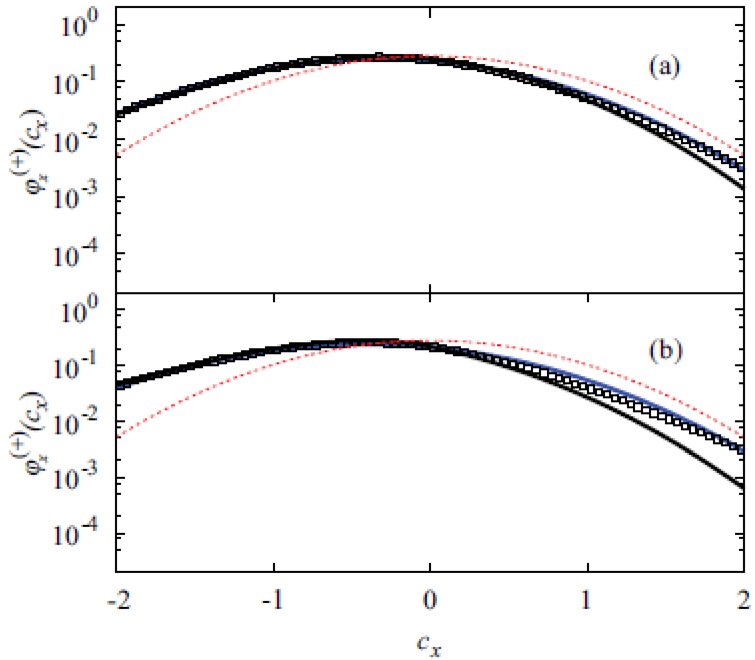


FIG. 7. (Color online) Logarithmic plots of the marginal distribution function $\varphi_x^{(+)}(c_x)$, as defined in Eq. (B15). Two cases are represented here: (a) $\alpha = 0.9$, $\gamma^* = 0.1$ and (b) $\alpha = 0.5$, $\gamma^* = 0.1$. The black and blue solid lines are the theoretical results derived from the BGK model and the ME formalism, respectively, while the symbols represent the simulation results. The red dotted lines are the (local) equilibrium distributions.

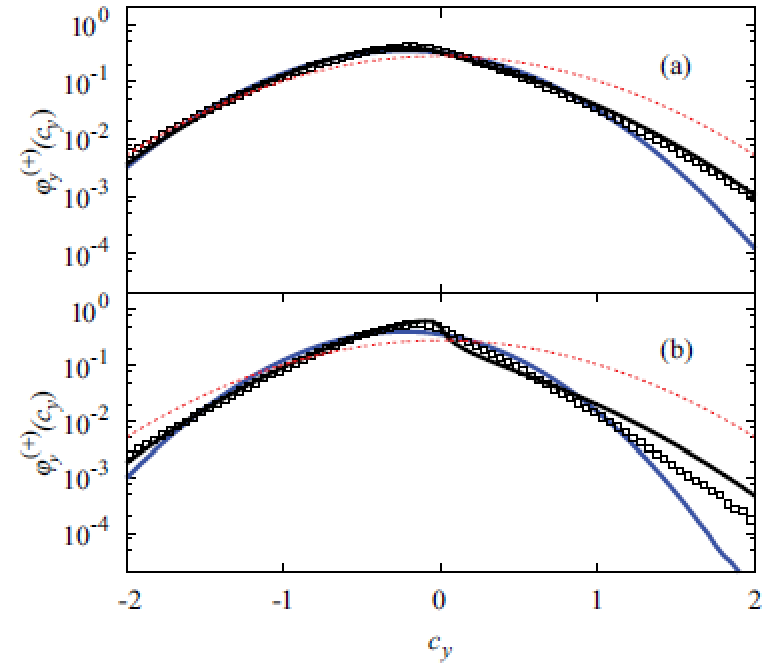


FIG. 8. (Color online) Logarithmic plots of the marginal distribution function $\varphi_y^{(+)}(c_y)$, as defined in Eq. (B16). Two cases are represented here: (a) $\alpha = 0.9$, $\gamma^* = 0.1$ and (b) $\alpha = 0.5$, $\gamma^* = 0.1$. The black and blue solid lines are the theoretical results derived from the BGK model and the ME formalism, respectively, while the symbols represent the simulation results. The red dotted lines are the (local) equilibrium distributions.

Many thanks for your attention !!!