# Gravity waves in water covered by a viscous layer and disk-like impurities

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# Introduction

Propagation of ocean waves can often be affected by the presence of materials on the water surface:

- oil slicks and floating debris

- grease ice (very thin, soupy surface layer) and pancake ice (disk-shaped ice blocks floating on the grease ice layer)

These situations lead to wave attenuation (Marangoni effect)

Waves can be used as probes to determine sea-ice proprieties. $\rightarrow$ Use remote sensing to measure modifications in wave propagation, and hence infer the ice thickness

# Introduction

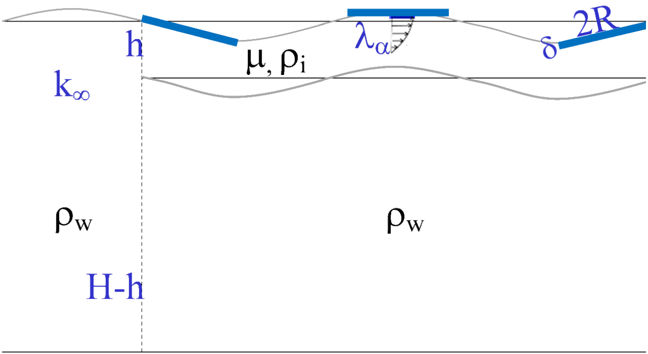
Theoretical understanding of how gravity waves are affected by sea ice is required.

Several models of wave propagation in ice covered oceans have been proposed where grease ice is treated as a viscous medium *(Weber 1987, Keller 1998)*.
Some generalizations of the viscous layer model provide:

- The effect of an eddy viscosity in the water column *(De Carolis & Desiderio 2002)*
- A viscoelastic component in the ice layer *(Wang & Shen 2010)*

Aim: Extend such models to account to effects of pancake ice.

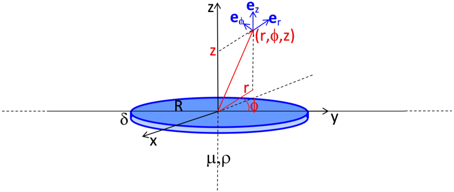
# Flow Scheme




Hypothesis:

$\delta, \lambda_a << R << k_\infty^{-1}$, $\quad H >> R$, $\quad k_\infty h << 1$

# Flow Perturbation by a single disk




Boundary conditions:

- $r < R, z = 0 \Rightarrow$
  - No slip,
  - Impermeability,
  - Zero total vertical forces ($\delta << R$)
- $r > R, z = 0 \Rightarrow$

  $\tau_{z\phi} = \tau_{zr} = \tau_{zz} = 0$

Perturbation expansion in term of:

$$\epsilon_k = k_\infty R, \quad \epsilon_\alpha = \lambda_\alpha / R$$

# Dilute theory

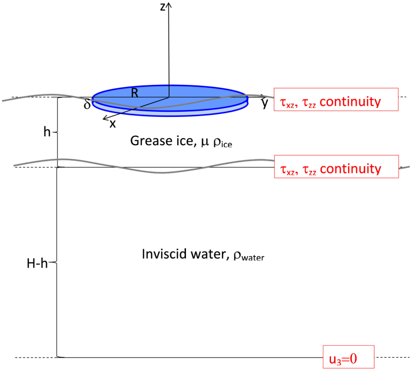
At macroscopic scale, the cumulative effects of the disks evaluated by means of a local spatial average.

$$\langle \tau_{xz} \rangle = \zeta \alpha \partial_x^2 \bar{U}_x, \quad \langle \tau_{zz} \rangle = \frac{i\sigma}{\omega} \partial_x^4 \bar{U}_z$$

$$\psi = \frac{h}{\lambda_\alpha}, \quad \hat{\alpha} = \alpha \lambda_\alpha = \sqrt{-i}$$

$$\zeta = \frac{11 f \nu R^2}{64} \tanh(\hat{\alpha}\psi), \quad \sigma = \frac{fgR^4}{64}$$

where $f$ the surface fraction of the disks




No viscoelastic constitutive law

# Dispersion relation

$$\hat{k} = \frac{k}{k_\infty}; \quad \hat{\alpha}_k = \sqrt{-i + \tilde{\nu}\hat{k}^2}; \quad \tilde{h} = k_\infty h; \quad \hat{H} = k_\infty H;$$

$$\hat{\rho} = \frac{\rho}{\rho_w}; \quad \hat{\zeta} = \frac{11\xi f}{64}\tanh(\hat{\alpha}\psi); \quad \hat{\sigma} = \frac{\xi^2 f}{64}$$

$$\psi = h/\lambda_\alpha = O(1), \qquad \tilde{\nu} = (\epsilon_k \epsilon_\alpha)^2 = (\lambda_\alpha k_\infty)^2 << 1,$$

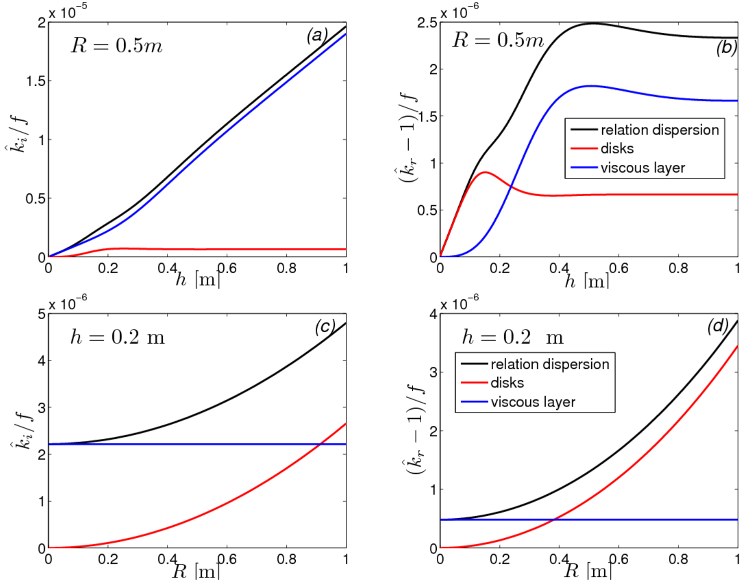
$$\xi = \epsilon_k/\epsilon_\alpha = R^2/(\lambda\lambda_\alpha) = O(1)$$

For $H \to \infty$, perturbative expansion up to $O(\tilde{\nu}^{(3/2)})$

$$\hat{k} \simeq 1 + \underbrace{\tilde{\nu}\hat{\rho}[i\hat{\alpha}\hat{\zeta} + \hat{\sigma}]}_{\text{main disks contribution}} + \underbrace{\tilde{\nu}^{3/2}\psi 8\hat{\rho}\left[i + \frac{\hat{\alpha}}{\psi}\frac{\cosh\hat{\alpha}\psi - 1}{\sinh\hat{\alpha}\psi}\right]}_{\text{viscous layer model}}$$

$$\underbrace{+\tilde{\nu}^{3/2}\psi\left\{2\hat{\rho}(1-\hat{\rho})\hat{\sigma} + \left[2\hat{\alpha}\hat{\rho}(1-\hat{\rho}) - \frac{4\hat{\rho}}{\psi}\frac{\cosh\hat{\alpha}\psi - 1}{\sinh\hat{\alpha}\psi}\right]i\hat{\zeta} - \frac{\hat{\rho}\hat{\alpha}\cosh\hat{\alpha}\psi}{\psi\sinh\hat{\alpha}\psi}\hat{\zeta}^2\right\}}_{\text{highest order correction}}$$

$$\nu = 10^{-2} m^2/s, \quad k_\infty = 0.06\ m^{-1}, \hat{\rho} = 0.917$$



# Close Packing

- In real pancake ice conditions the surface fraction $f$ is $O(1)$
- Pancakes are not free to move horizontally $\Rightarrow$ they will forced to follow neighboring pancakes in their motion.

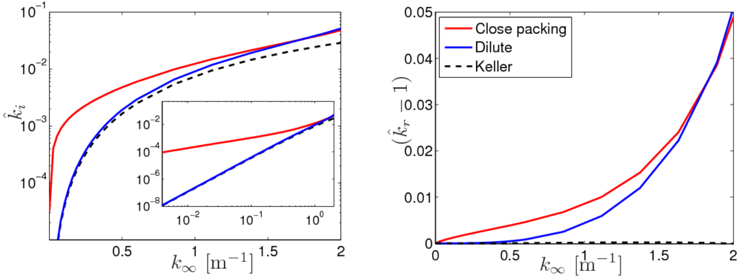
No-slip boundary condition applied to islands of pancakes of horizontal size $\hat{R} \sim 1/k$, rather than to individual pancakes.

$$\tau_{xz} = \frac{c\mu\alpha}{k^2}\partial_x^2 \bar{U}_x \tanh(\hat{\alpha}\psi), \quad z = 0 \;\Rightarrow\; \hat{R} \approx 0.39\sqrt{c}\lambda$$

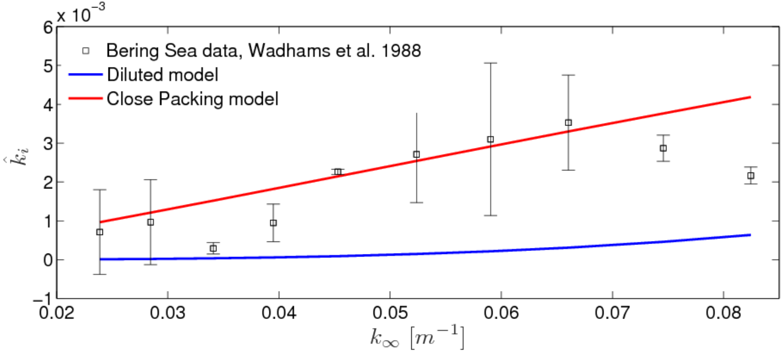
$$\frac{\hat{k}}{\hat{k}_H} \simeq 1 + \tilde{\nu}^{1/2} \frac{i\hat{\alpha}c\hat{\rho}\hat{k}_H^2 \tanh(\hat{\alpha}\psi) + (1-\hat{\rho})(1+c)(\hat{k}_H^2 - 1)\psi}{(1+c)[1 + (\hat{k}_H^2 - 1)\hat{H}]}$$

where $\hat{k}_H$ *s.t.* $\hat{k}_H \tanh(\hat{k}_H \hat{H}) = 1$

$\nu = 10^{-2} m^2/s,\ \hat{\rho} = 0.917,\quad h = 0.5\ m,\quad R = 0.6\ m,\quad c = 0.2$



# Comparison with Field Data: ice floes




$$\nu = 0.01\ m^2/s,\ \hat{\rho} = 0.917,\ R = 10\ m,\ h = 0.16\ m,\ f = 1$$

$$c = 1.3 \Longrightarrow \hat{R} = 0.44\lambda \Longrightarrow 3R < \hat{R} < 12R$$

# Concluding remarks

- $\epsilon_k, \epsilon_\alpha \ll 1 \Rightarrow$ disk of the size of pancakes up to small floes
- For long waves, the locking mechanism produces orders of magnitude increase of both damping and dispersion with respect to artificially setting $f = 1$ in the dilute model
- Close packing model reproduce, at least as orders of magnitude, field data with reasonable values of viscosity
- Importance of the viscous layer contribution to the disk dynamics also when the layer is very thin

Future work:
-Take into account the interactions between pancakes: collision, non-linear elasticity,..
-effect on the dynamics of a disk thickness comparable to the depth of the viscous boundary layer