Modelling of Residence Time Dispersion of Falling Droplets in Spray Dryers

Kevin Cronin[a] & Song Miao[b]

[a]Department of Process & Chemical Engineering, University College Cork

[b]Teagasc, Food Research Centre (Moorepark)

Agriculture and Food Development Authority

Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

Residence Time Dispersion

Spray Drying

Spray Dryer: Liquid feed is atomised, sprayed in at the top and falls through the chamber while in contact with a hot gas. Liquid droplets converted to solid powder particles.

Two variable parameters:

1.Initial droplet size distribution

2.Residence time distribution

Hence obtain a dispersion in final moisture, temperature and quality. ('large droplets with short residence times come out wet while small droplets are overdried')

The ideal is mono-dispersed sized droplets wth identical residence times.

Dryer Performance Analysis (at early design stage)

Having a probabilistic model of residence time and droplet size

Evaporation Model

$$m_s \frac{dx}{dt} = h_m \pi d^2 \left(\rho_{v,s} - \rho_{v,a}\right)$$

Heat Transfer Model

$$m c_p \frac{dT}{dt} = \Delta H_v m_s \frac{dx}{dt} + h_t \pi d^2 \left(T_a - T\right)$$

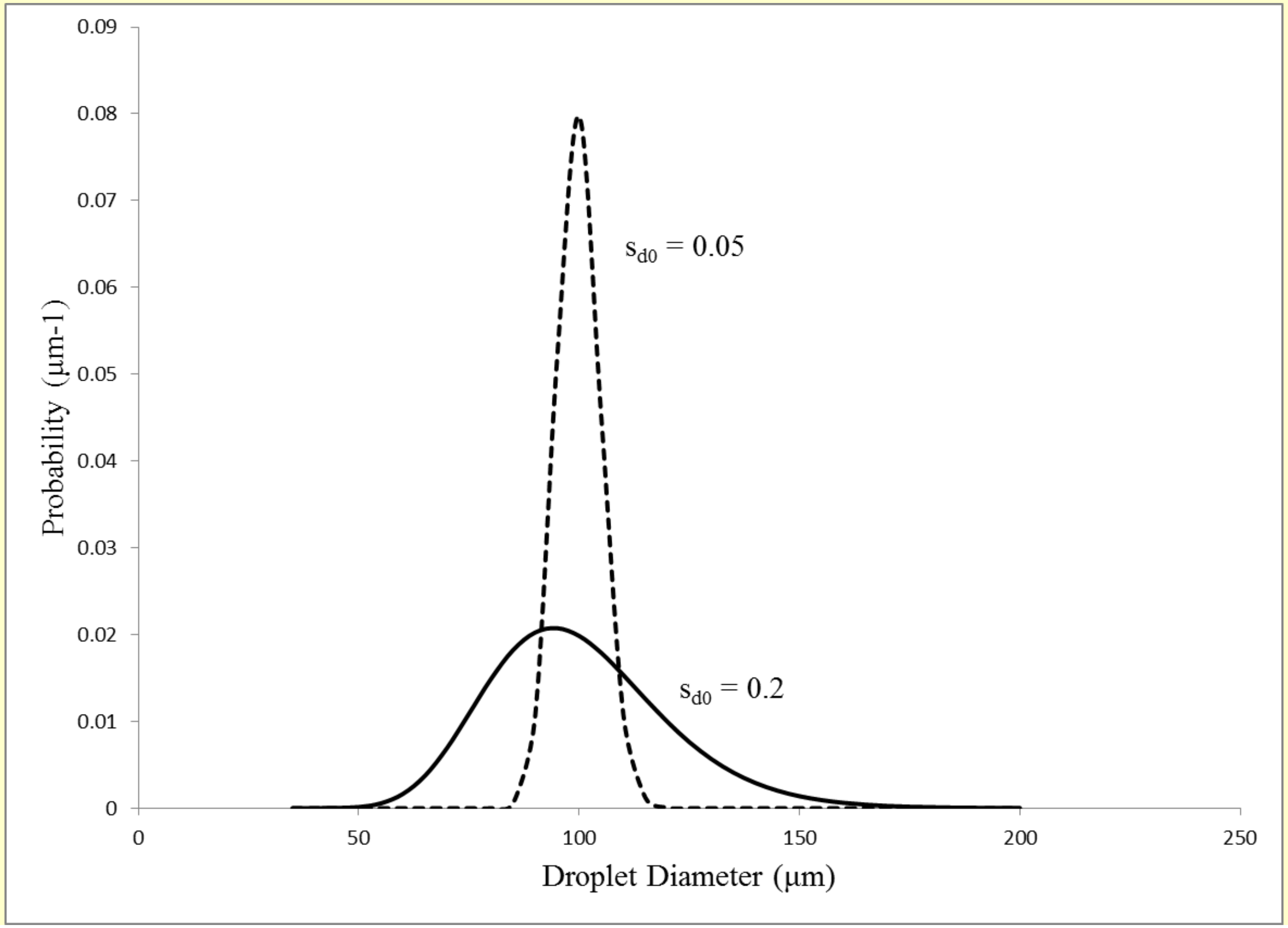
Final Moisture Content

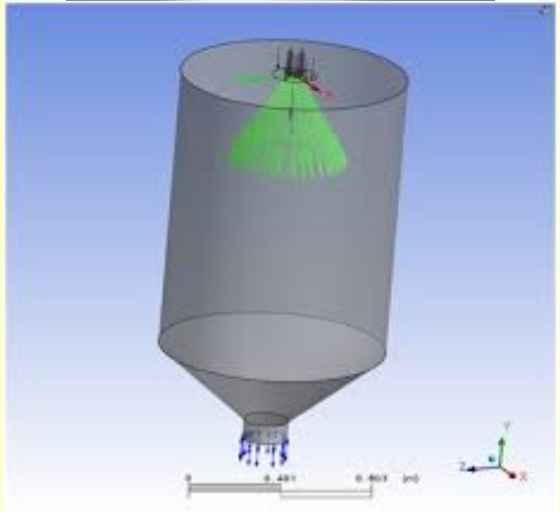
$$x_f = \left(x_c - x_e\right) e^{\frac{c_2}{c_1}(x_0 - x_c)} e^{-c_2 \frac{\tau}{d_0^2}} + x_e$$

Residence Time Dispersion

Initial Droplet Size Distribution

Initial droplet diameter is approximately centred about 100 um, right skewed and ranges from 10 um up to 200 um (Log-Normal distributed).





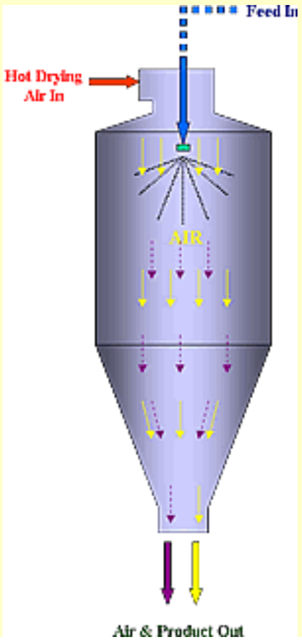
Residence Time Dispersion

Residence Time

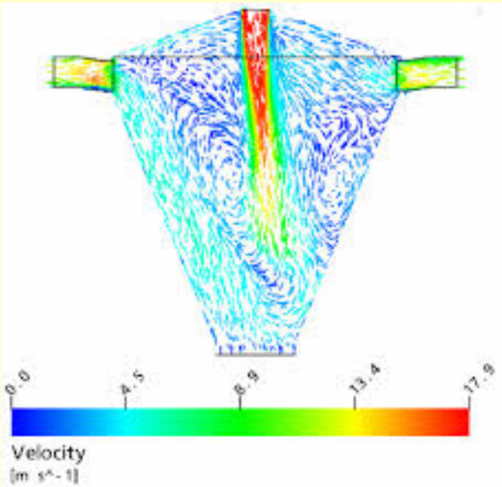
Droplets poly-dispersed in size falling through a co-current air stream; residence time occurs when particle vertical displacement equals chamber height, H.

Very complex to determine unless you know the entire velocity and trajectory history (spatial pattern of air flow, swirl and turbulence, recirculation, inter-particle collisions etc.). All dryer specific.

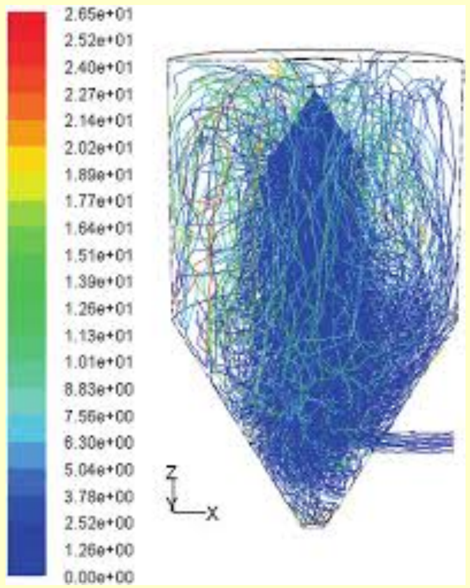
Schematic



Air Flow (Gabites et al.)



Droplet Trajectories (Anand. et al.)



Residence Time Distribution

Experimentally it is found that for many systems:

- Residence time distribution is also right-skewed (average values can vary 1 s up to 100s)
- Heavier droplets/particles have shorter residence times (gravity settling)
- Heavier particles exhibit less dispersion in residence time (less sensitive to gas flow fluctuations)
- While mean residence time and dispersion in residence time are dryer specific, coefficient of variation in residence time (standard deviation in residence time divided by the mean value) is about 0.3 to 0.4 for most reported systems.

Simple Approach to Residence Time and Droplet Diameter

Examine a single droplet; Gravity and drag force dominate droplet kinetics.

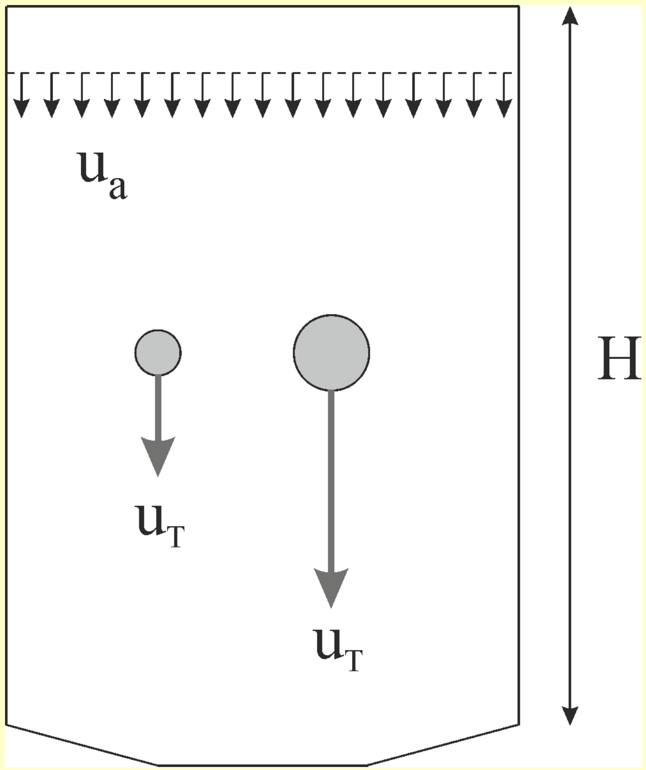
Pressure nozzle in a tall form spray dryer with a plug flow zone.

Assuming droplets decelerate rapidly to terminal velocity in the uniform co-flow gas stream

$$\tau = \frac{H}{u_T + u_a}$$

Terminal velocity depends on diameter and the drag coefficient (diameter is time-invariant)

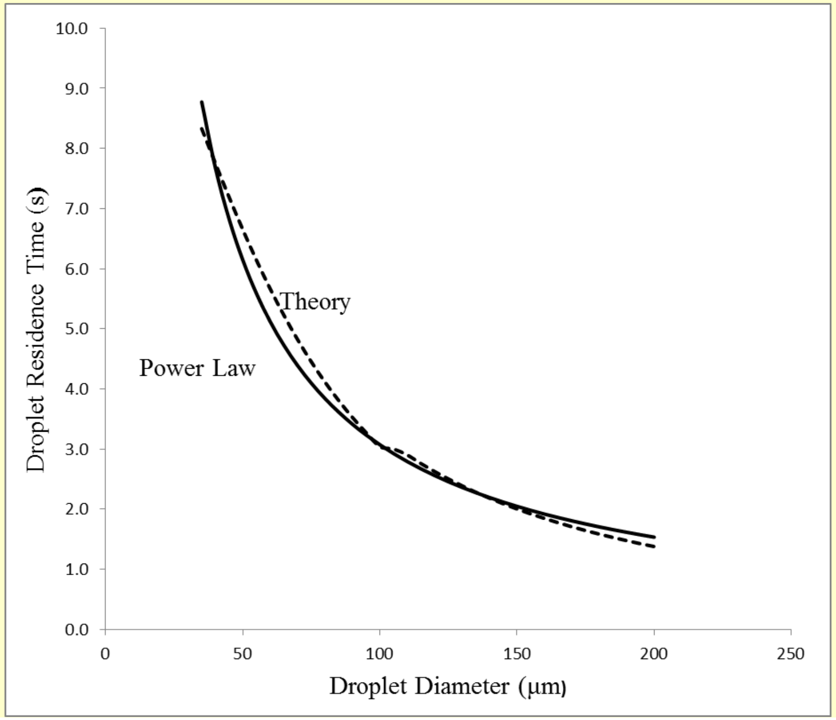
$$u_T = \left(\frac{4d\rho_p g}{3C_D \rho_a} \right)^{0.5}$$



Deterministic Relation between Residence Time and Droplet Diameter

For droplets within a certain size range, the relationship between residence time and droplet diameter can be approximated with a power law relationship

$$\tau = \frac{H}{B_1 \, d_0^{\,B_2}}$$



B_1 depends on air and particle physical properties and B_2 is equal to 1.

Probabilistic Droplet Kinetics

Because of the real complexity of particle-fluid interaction [Mass (evaporation), Momentum (drag), Thermal (heat transfer)], there is not a simple deterministic relationship between τ and d_0.

Introduce the parameter, q:

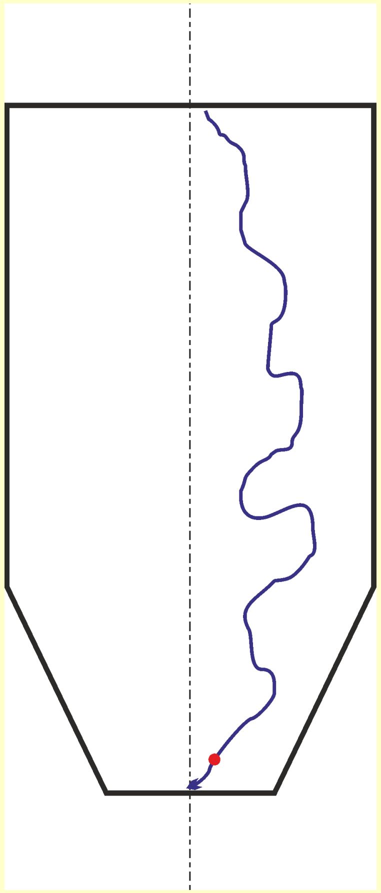
$$\tau = \frac{H}{B_1 d_0} q$$

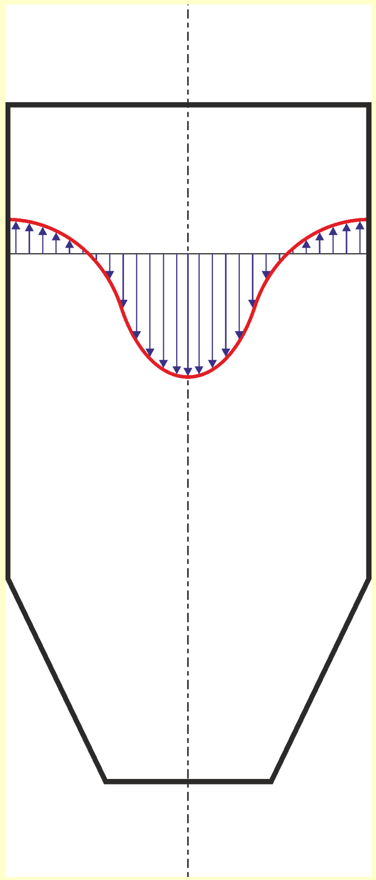
where q is termed a kinetic dispersion parameter and is a stochastic term reflecting the many random inputs to droplet motion. It is a measure of how the dryer chamber promotes or affects variability in residence time.

Probabilistic Droplet Kinetics

Expressions for q can be obtained depending on its physical origin:

- Fluid turbulence / Random eddies
- Systematic variation in the spatial fluid pattern

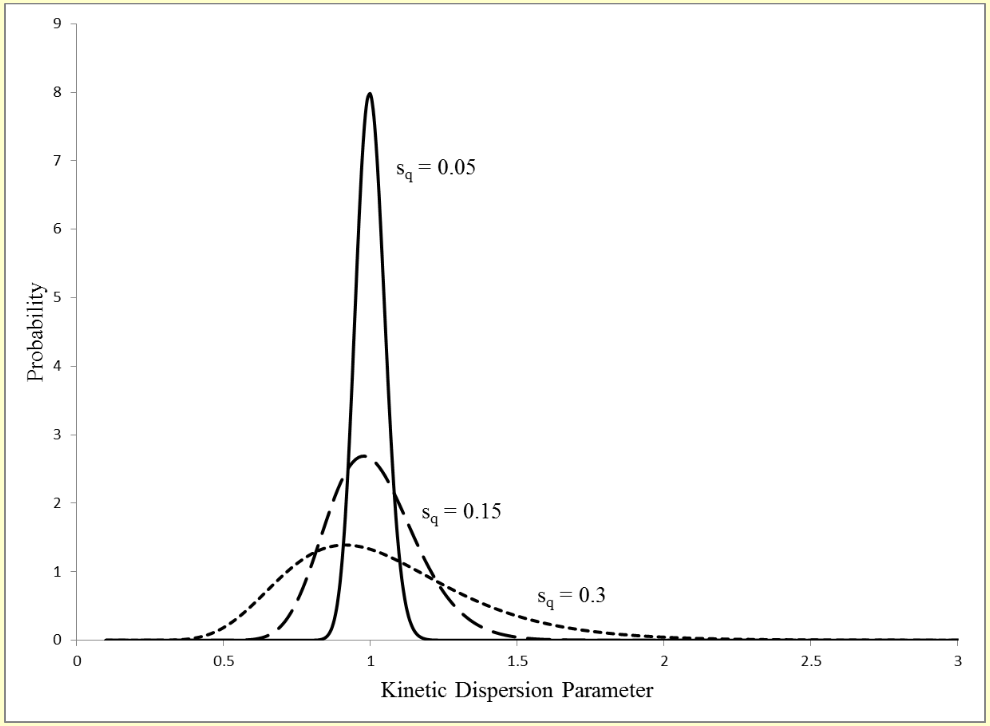




Kinetic Dispersion Parameter

It can be shown that q can be represented as a log-normal distributed random variable (corresponding Normal distribution has zero mean, variance s_q^2).

q is always positive and as s_q increases in magnitude, it varies more widely about the central value of 1.



Droplet Residence Time & Diameter

Residence time will also be Log-Normally distributed as itself being equal to the ratio of two log-normally distributed variables q and d_0

Droplet residence time and diameter are represented by the bivariate log-normal distribution as

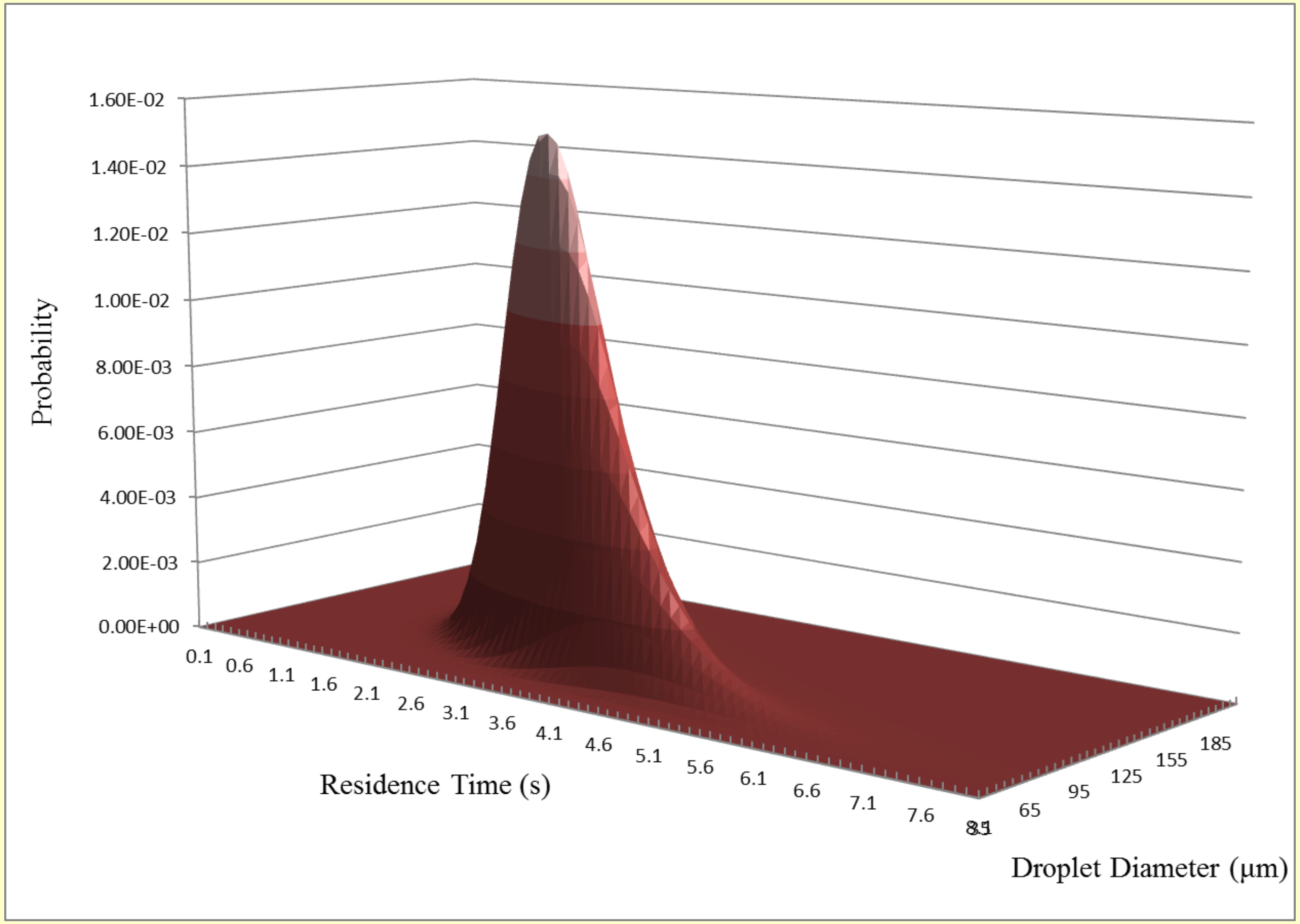
$$p(d_0, \tau) = \frac{1}{2\pi s_{d_0} s_\tau \sqrt{1-\zeta^2}} \frac{1}{d_0 \tau}$$

$$\exp\left[-\frac{1}{2(1-\zeta^2)}\left\{\left(\frac{\ln d_0 - m_{d_0}}{s_{d0}}\right)^2 + \left(\frac{\ln \tau - m_\tau}{s_\tau}\right)^2 - 2\zeta\left(\frac{\ln d_0 - m_{d_0}}{s_{d0}}\right)\left(\frac{\ln \tau - m_\tau}{s_\tau}\right)\right\}\right]$$

Five parameters for the distribution: m_{d0}, s_{d0}, m_τ, s_τ and ξ.

Droplet Residence Time & Diameter

Joint probability density function.



Statistics of Droplet Residence Time & Diameter

Knowing the mean and standard deviation in droplet diameter, the parameters of the marginal distribution of droplet diameter, m_{d0} and s_{d0} can be obtained.

$$m_{d_0} = \ln\left[\frac{\mu_{d_0}^2}{\sqrt{\mu_{d_0}^2 + \sigma_{d_0}^2}}\right] \qquad s_{d_0} = \sqrt{\ln\left[1 + \frac{\sigma_{d_0}^2}{\mu_{d_0}^2}\right]}$$

The parameters for residence time will be

$$m_{\tau} = \ln\left(\frac{H}{B_1}\right) - m_{d_0} \qquad s_{\tau}^2 = s_{d_0}^2 + s_q^2$$

Standard deviation in residence time is given as

$$\sigma_{\tau} = \sqrt{\left(\exp\left[s_{\tau}^2\right] - 1\right)\exp\left[2m_{\tau} + s_{\tau}^2\right]}$$

Dispersion in residence time depends on magnitudes of s_{d0} and s_q.

Statistics of Droplet Residence Time & Diameter

The correlation coefficient, ξ between residence time and droplet diameter is

$$\zeta_{\tau,d} = \frac{\exp\left[\rho\, s_{d_0}\, s_{\tau}\right] - 1}{\sqrt{\left(\exp\left[s_{d_0}^{2}\right] - 1\right)\left(\exp\left[s_{\tau}^{2}\right] - 1\right)}}$$

showing that as the magnitude of the standard deviation in the kinetic dispersion parameter, s_q increases, the level of correlation between residence time and droplet diameter falls.

As s_q is raised, random dispersion in residence time increases and as a corollary, an decreasing proportion of the variability in residence time is due to the variability in droplet size.

Residence Time Dispersion

Simulation Data

Initial droplet velocity	u_0	3 m/s
Initial droplet diameter	d0	100 um
Air temperature	T_a	90 ºC
Air velocity	u_a	0.1 m/s
Chamber height	H	1.1 m
Residence time constant parameter	B_1	3582 s^{-1}
Residence time power parameter	B_2	1

Parameter Studies & Output

The approach enables the relative influence of dispersion in droplet size (reflecting atomiser characteristics and measured by s_{d0}) and dispersion in airflow patterns (reflecting dryer chamber configuration and operation and measured by s_q) on residence time.

1. Mean residence time, μ_T increases slightly (by about 10 %) with higher levels of droplet size dispersion and randomness in the kinetic parameter.

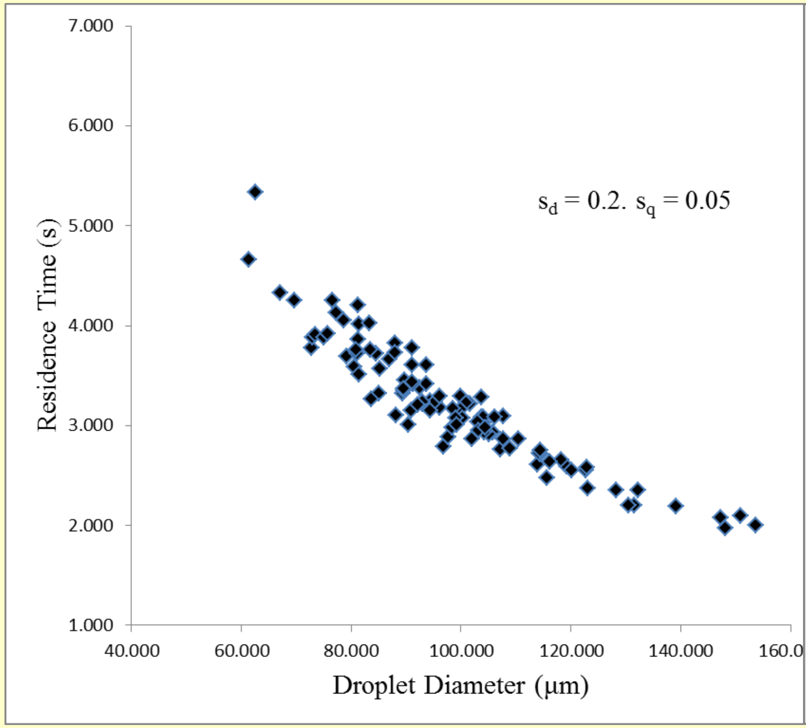
2. Standard deviation in residence time, σ_T increases significantly as both s_{d0} and s_q increase and both are of approximately equal importance.

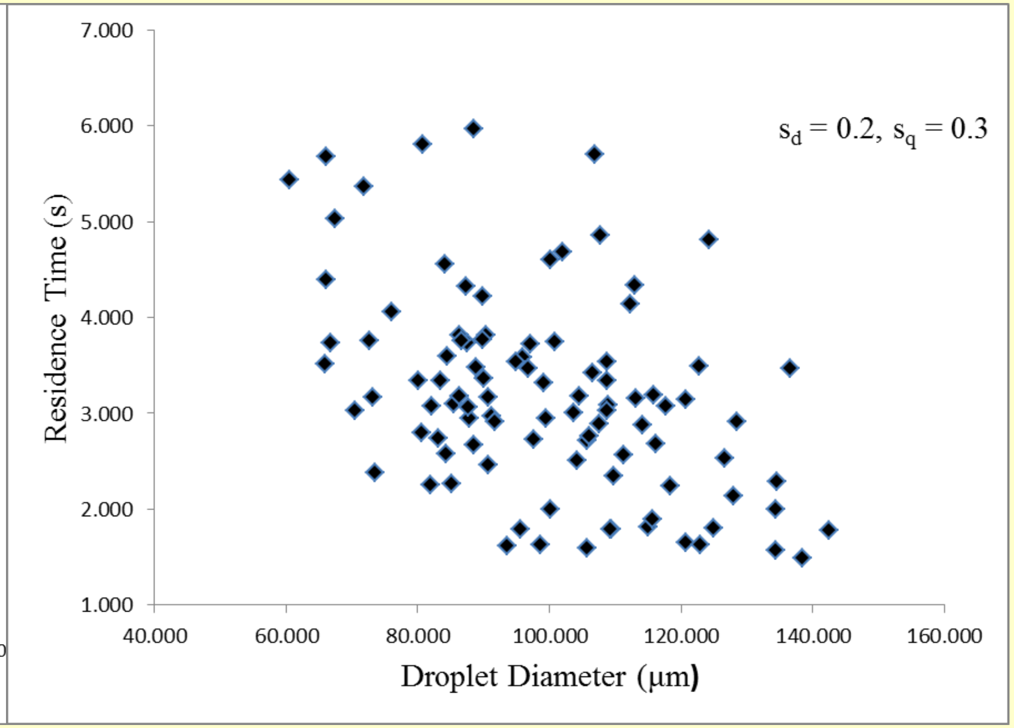
3. Correlation (negative) between droplet residence time and droplet diameter is pronounced for low values of s_q but falls towards zero as airflow pattern dispersion rises.

Residence Time Dispersion

Scatter Plots of Residence Time Dispersion and Diameter

Much larger scatter in the relationship between them is clearly evident as the s_q parameter is raised.

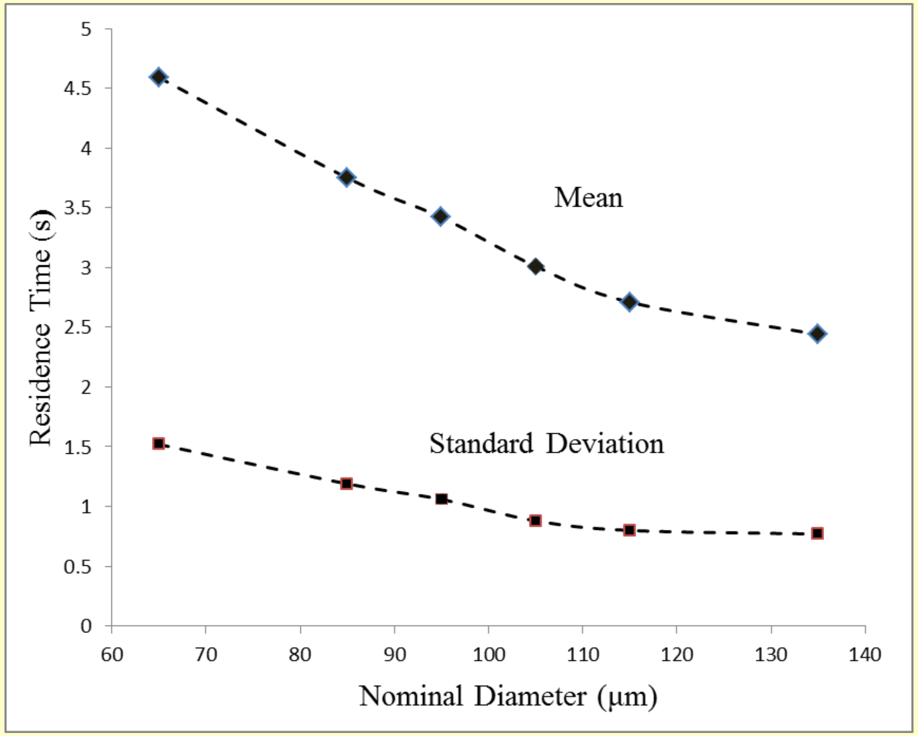




Residence Time and Droplet Diameter

Mean residence time falls with an increasingly large nominal diameter.

Standard deviation in residence time falls as mean diameter increases i.e. larger droplets tend to have a residence time closer to their nominal mean residence time than smaller ones.



Obrigado pela vossa atencao.

Perguntas ? (in English please)