Class of periodic and quasiperiodic trajectories of particles settling under gravity in a viscous fluid

Maria L. Ekiel-Jeżewska

Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw, Poland


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The main idea

Many-particle systems in fluids

- instabilities
- hydrodynamic clustering
- chaotic scattering
The main idea

Many-particle systems in fluids

- instabilities
- hydrodynamic clustering
- chaotic scattering

→ periodic oscillations
Sedimentation (example: a drop of cream into water)

- suspensions
- suspension drops
- benchmark: three particles (Janosi, Tel et al. 1997)
Suspension drops settling at a low Reynolds number

- particles are more dense than the fluid
- the same fluid inside/outside the drop
- initially spherical shape, random distribution of particles
- experiments + numerical simulations: Batchelor, Nitsche, Schaflinger, Machu, Ladd, Metzger, Guazzelli, Bosse, Koch, ...
How the drop evolves?

Characteristic features

- PARTICLES CIRCULATE inside the drop
- some of them are lost in a TAIL
- DROP DEFORMS - expands, flattens, forms a torus
- DROP BREAKS, with a wide range of destabilization times

Why?

Point-particles, Myłyk, Meile, Brenn, MEJ (2011)
To be understood

- CIRCULATIONS
- TAIL
- DEFORMATION
- DESTABILIZATION
An idea how to model a suspension drop

Hocking's periodic orbits of 4 particles (1964)

Movies: M. Sikora + MEJ, hydro.ippt.gov.pl
Observer does not move
Observer moves with the drop center-of-mass
Choosing partners

Two pairs: red-green and light-dark blue
Sensitivity to a tiny change of the initial conditions

Exchanging partners!
Family of periodic motions

Initially point particles form a rectangle
Family of periodic motions

Initially point particles form a rectangle

A better shape?
Suspension drop: a better model

A regular right prism

16 point particles
Trajectories with respect to center of mass

c=2.1
Destabilization

\[ c = 0.8 \]
Destablization

c=0.8
CIRCULATIONS

TAIL-
statistical fluctuations → particles which are too far from periodic orbits stop circulating and are left behind the drop as a tail

DEFORMATION

DESTABILIZATION
Once upon a time there was a Drop and a Tail...
How the tail interacts with the drop?

Can a small tail change motion of a large drop?
Evolution of the drop shape caused by a tail

Regular prism: 16 point particles; Tail: 1 point particle

c=0.9
Hydrodynamic interaction induced by the tail

Streamlines generated by a single point force

The resulting velocities of point-particles

Particles inside the drop
- are repelled horizontally from the drop center
- are attracted vertically to the centerplane
The analogy

decrease of $c$
in the regular prism ↔ time evolution of suspension drop

How the dynamics of regular prisms depends on $c$?
Trajectories with respect to center of mass

\[ c = 2.1 \]
Trajectories with respect to center of mass

c = 1.6
Trajectories with respect to center of mass

\[ z_{CM} \]

\[ y_{CM} \]
Trajectories with respect to center of mass

c=0.4
Trajectories with respect to center of mass

\[ c = 0.1 \]
Trajectories with respect to center of mass

c=0.05
Trajectories with respect to center of mass

$c=0.047$

[Graph showing trajectories with axes labeled $y_{CM}$ and $z_{CM}$]
Long and flat CM trajectories
Long and flat CM trajectories

c=0.05
Long and flat CM trajectories
Length $L$ of a center-of-mass trajectory and period of oscillations $T$ increase when $c$ decreases.

Is there a critical value of $c$?
Critical value of $c$

\[ T = T_0(c - c_0)^{-\alpha}, \quad c_0 = 0.044788..., \quad \alpha = 1.5 \]
Critical value of $c$

\[ T = T_0 (c - c_0)^{-\alpha}, \quad c_0 = 0.044788..., \quad \alpha = 1.5 \]

But suspension drops are not as thin!
Conclusions

A new class of periodic motions

- found and analyzed
- used to progress in understanding how suspension drops evolve

- **CIRCULATIONS → DESTABILIZATION**
  Unstable periodic orbits break at a time $\tau_d$ (wide range)

- **TAIL → DROP DEFORMATION**
  Particles are left behind the drop in a tail if sufficiently separated from “periodic orbits”. The tail repels particles horizontally from the drop center and attracts them vertically to the centerplane, at a time scale $\tau_t$

- $\tau_t \gg \tau_d$, separation of time scales
Outlook

Are there periodic solutions

▸ for systems with elastic constraints?

▸ for more particles?

▸ for out-of-phase initial configurations?

▸ (with symmetrization)
Outlook

Are there periodic solutions

- for systems with elastic constraints?
- for more particles?
- for out-of-phase initial configurations?
- (with symmetrization)

Marek Bukowicki
Tuesday 17:15

Marta Gruca
Wednesday 9:45